

Numerical Methods - Lecture 1

Topics:

Numerical Methods Introduction

WHY ARE WE STUDYING THIS???

matrix methods

curve fitting

root finding

optimization

statistical analysis

probability analysis

integration / differentiation

ordinary differential equations

signal processing

NUMERICAL METHODS (why?)

Our creations are based on mathematical models of what occurs in nature.

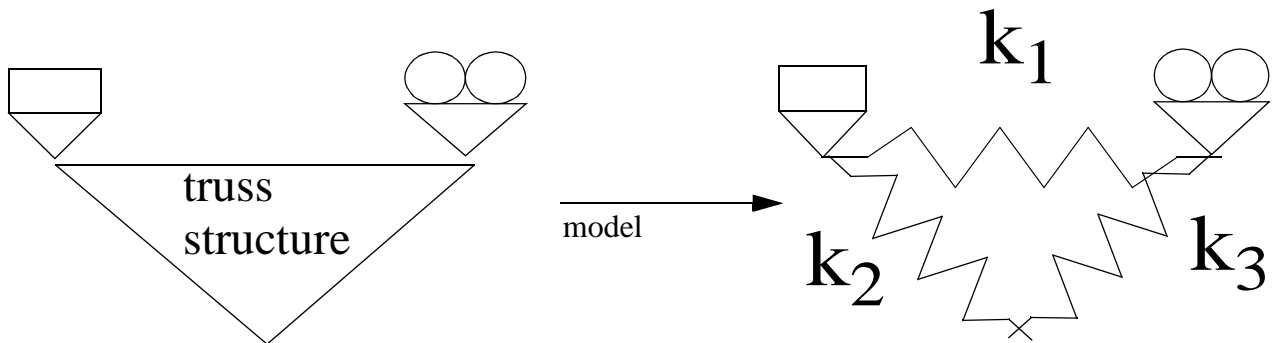
- Structural response to wind/waves/earthquakes
- Airplane performance
- Chemical reactions
- Automobile suspensions
- Steel bridge deformation under heavy railroad loads
- Traffic patterns for infrastructure design
- Maximum capacity of piles holding up foundations
- Waste water treatment plants
- The betting line on Florida vs. Florida State
- Insurance premiums
-

These mathematical models of physical processes allow us to design and build new creations that account for the various stresses they will see. Without numerical methods (the creation and application of mathematical models), there would be no aircraft, tall buildings, long bridges, etc.

We'll look at a brief introduction of each of the topics to be covered.

Matrix methods - solving simultaneous equations

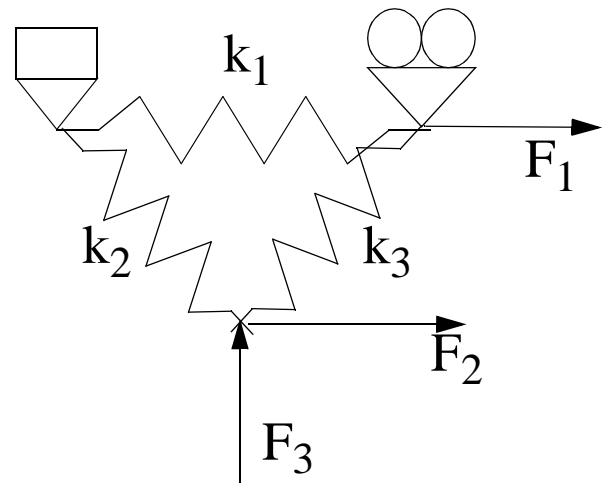
- We're familiar with the equation relating force and displacement for a spring as $F = k * X$.
- We've also learned how to analyze a simple truss structure in statics
- All a truss is permitted to do is stretch or compress. So one way of analyzing truss type structures is to visualize the structure as a group of attached springs. For example, the simple truss below can be represented as springs.



- Usually engineers know what the constant k is for each spring, and the forces F acting on the structure.
- If a force pushes or pulls on the end of one spring, it affects the other springs also. This results in what's called a 'coupled' system. That is, what happens to one truss member may affect another.
- With a little work (CES 4141) we can find three coupled equations that describe the behavior of this system (how much does each spring stretch when we apply forces ?).
- Each equation uses the spring concept $F = k * X$. The unknown displaced position of the ends of each spring (X) after the forces are applied are then found by solving a system of linear equations.

$$F = k * X$$

$$\begin{aligned} 3k_1 X_1 - 0.5k_2 X_2 - k_3 X_3 &= F_1 \\ -k_1 X_1 + 2.5k_2 X_2 - 0.5k_3 X_3 &= F_2 \\ -2k_1 X_1 - 0.5k_2 X_2 + 2.5k_3 X_3 &= F_3 \end{aligned}$$



- These equations can be solved for X_1, X_2, X_3 by using matrix methods of solving simultaneous equations.
- once we solve for the displacements X_1, X_2, X_3 , we can determine the internal forces in the truss and design it to withstand the forces.

Curve Fitting

finding mathematical functions $f(x)$ to describe observed behaviors
i.e. define a curve that fits some measured data

We will use the function $f(x)$ to PREDICT future behavior.

ANALYTICAL :

- Sometimes an exact solutions (equation) can be derived using known physical laws.
e.g. beam deflection using continuum mechanics

NUMERICAL MODELS :

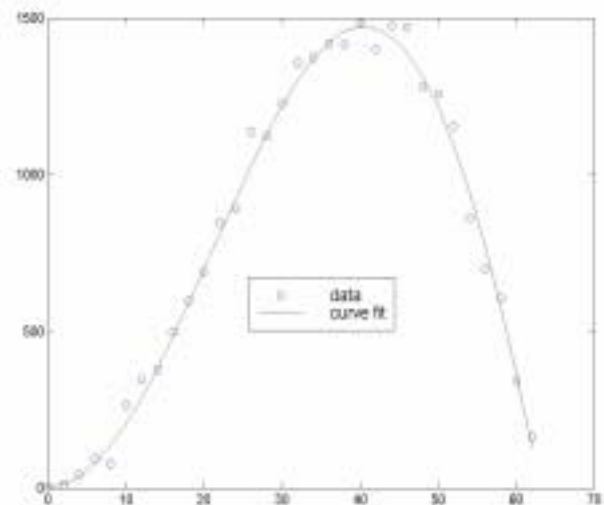
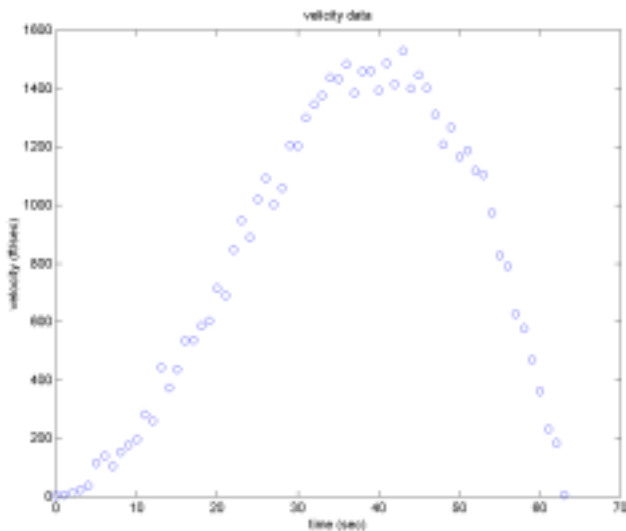
- Exact solution not known or too complex.
- Instead we approximate behavior by observing the process (taking data)
- Fit model to observed data.
e.g. force vs. displacement for steel
e.g. rocket velocity vs. time
- Recall the practice exam question about rocket performance with time

Let's say an equation for vertical velocity as a function of time is given as:

$$v(t) = 4 + 2.13t^2 - 0.0013t^4 + 0.000034t^{4.751}$$

Where did this equation come from? Analytical or numerical model?

Suppose the rocket was launched and velocity was measured (poorly), giving the plot on the left:



We then apply a numerical CURVE FITTING algorithm to the data. The outcome is the following equation

$$v(t) = 4 + 2.13t^2 - 0.0013t^4 + 0.000034t^{4.751} \quad \text{shown with the data on the right}$$

Root Finding

We now have the function (model) describing the rocket velocity as a function of time ($v(t)$)
What can we do with it?

- Use the model to construct / define solutions

Suppose we need to know when velocity will reach 500 feet/sec. There are two ways to solve this problem

Problem: using $v(t) = 4 + 2.13t^2 - 0.0013t^4 + 0.000034t^{4.751}$, find the time at which velocity = 500 ft/sec.

Solution method #1: attempt to find an Analytical Solution

set the left hand side equal to 500 and solve for t $500 = 4 + 2.13t^2 - 0.0013t^4 + 0.000034t^{4.751}$, find t

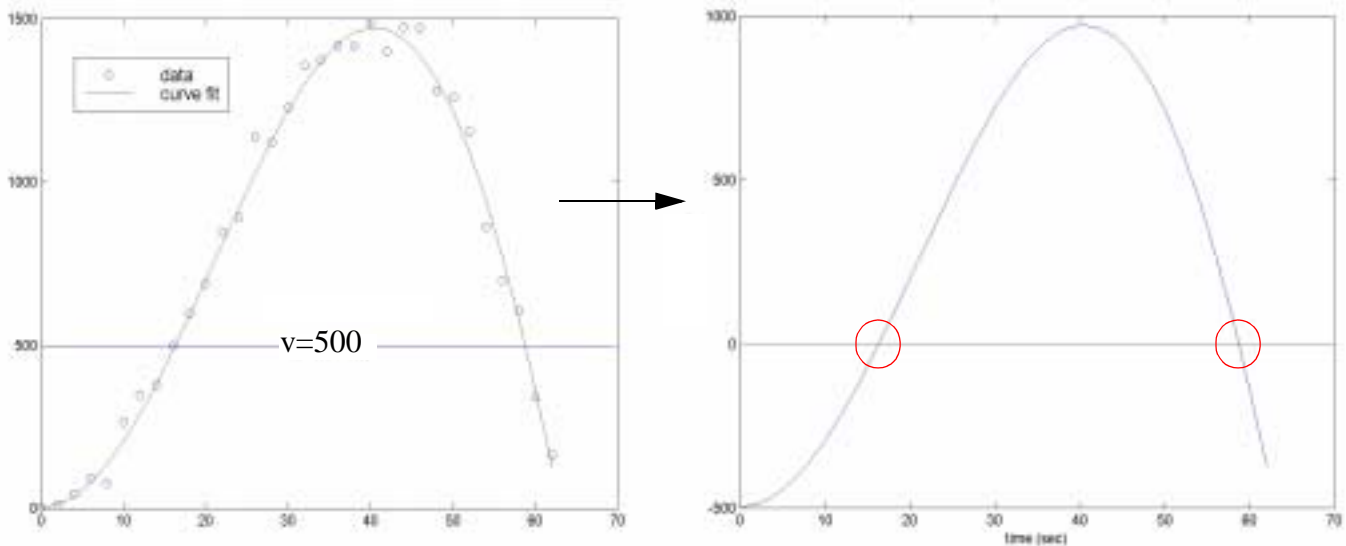
Solution method #2: use Numerical Root Finding Method to find when $v(t) = 500$

Recast to find when function is equal to zero

find t such that $v(t) - 500 = 0$

$$v(t) - 500 = 4 + 2.13t^2 - 0.0013t^4 + 0.000034t^{4.751} - 500 = 0$$

Now apply a ROOT FINDING method to identify where the curve crosses zero



Optimization - identifying minima / maxima

Suppose we need to know two more things about the model for rocket velocity:

Problem:

- 1) when the rocket will reach its maximum velocity?
- 2) what that maximum velocity will be?

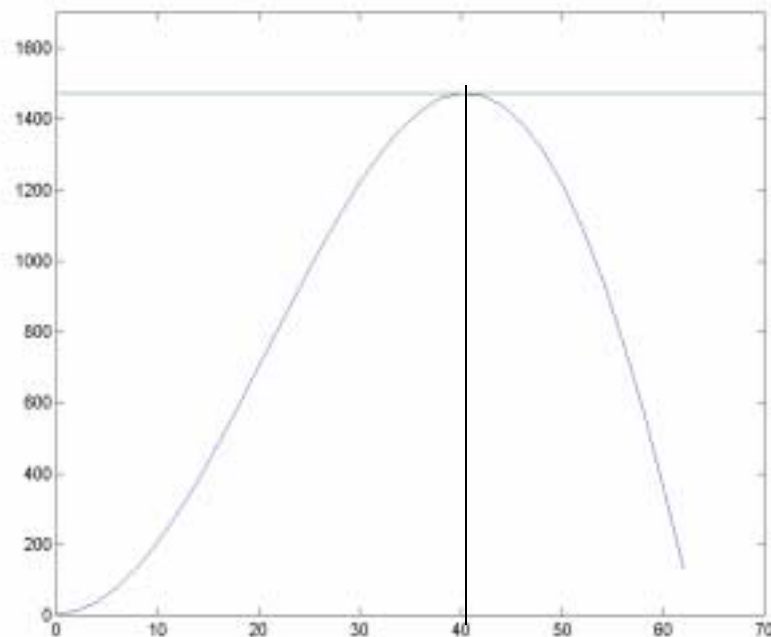
Solution Method #1: Analytical Solution

We can find the maximum of a function using calculus

Take the derivative of the function, set it to zero, and solve for t

Solution Method #2: Numerical Optimization Method to find the maximum of a curve

Optimization algorithm will locate the maximum value and the time it occurs



Integration / Differentiation

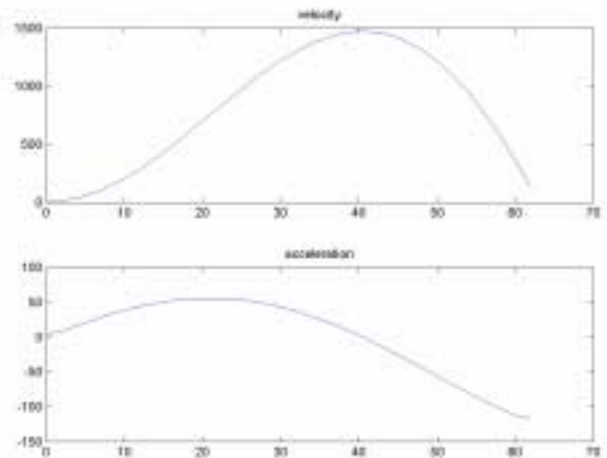
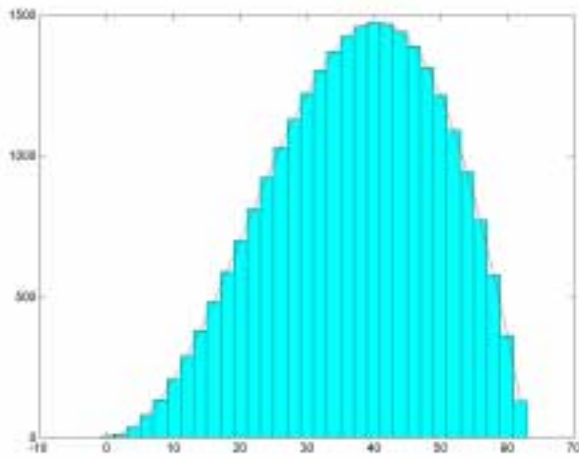
We have the velocity as a function of time $v(t)$

Suppose we want to estimate the rocket's height and acceleration also...

- Integration of vertical velocity gives us height
- Differentiation of vertical velocity gives us vertical acceleration

Again, perhaps its possible to find an analytical derivative and integration.

But if that is impossible or overly complex, we can use numerical methods to approximate the results.



Ordinary Differential Equations:

Given the differential of a function at any point, and initial conditions

Find the function itself

e.g. given $\frac{dy}{dx} = 2x$ and $y(x=0) = 0$

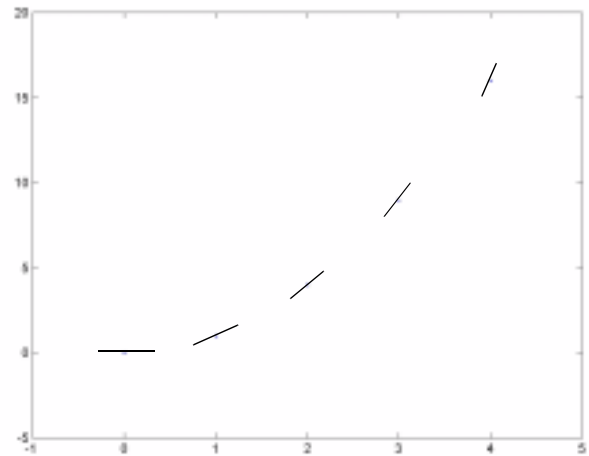
find the function $y(x)$

- All we have is the slope for any given x value
- We have to piece together the actual function

Analytical solution

or

numerical method



Real example:

building response to wind loads is described by a differential equation

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t)$$

Given the building parameters K , C , M find the building displacement x due to wind forces $f(t)$

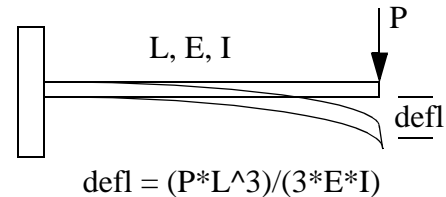
Probability / statistical analysis

The analyses of the behavior of systems should account in some manner for the uncertainties in the defining parameters.

For example, let's consider the cantilevered beam again. A point load is placed at the tip, and deflection is measured there.

We calculate that if the deflection exceeds 10% of the length L of the beam, then the system will break. That is,

$$\text{failure} = \text{defl} > 0.1 * L$$



Deterministic case:

If we are given constant, known values for E , I , L , and P , we can calculate deflection and say 'yes' or 'no' to the question of failure.

Uncertain case:

However, its not always true that we know the exact values for these parameters. What if P is only an estimate, how reliable is the estimate? Could it be off by 5% ? 20% ? What if E was an average value calculated from several batches of steel. Does this particular beam have a slightly higher or lower E value? How does this affect our ability to say 'yes' or 'no' to the failure question?

Probabilistic analysis:

The goal is to quantify the uncertainty in the given parameters.

Then we can then look at the deterministic question

Deterministic Q: Will the deflection exceed 10% of L ?

and re-phrase it to account for uncertainty:

Probabilistic Q: Given what we know about the uncertain quantities, what is the probability that deflection exceeds 10% of L ? This is no longer a 'yes' or 'no' question.

This represents the LIKLIHOOD of the beam failing given the uncertain information available.

This is also called **Reliability analysis**

We'll spend some time discussing how to define and then answer such questions