STIFFNESS BY DEFINITION

CES 4141

GOAL: IDENTIFY (DERIVE) STIFFNESS MATRIX $\underline{K}$ OF FRAMES + TRUSS SYSTEMS

PROPERTIES OF $\underline{K}$

1. FUNCTION OF SYSTEM GEOMETRY, MATERIAL PROPERTIES, AND BOUNDARY CONDITIONS
2. NOT A FUNCTION OF EXTERNAL LOADS
3. $\underline{K}$ ALWAYS SQUARE, SYMMETRIC MATRIX $(K_{ij} = K_{ji})$
4. DIAGONAL TERMS ALWAYS POSITIVE $K_{ij} = +$ FOR $i = j$

DEFINITION

> K_{ij} = FORCE (MOMENT) REQUIRED AT D.O.F. i TO HOLD A UNIT DEFLECTION AT D.O.F. j, WHILE HOLDING ALL OTHER D.O.F. = 0

ANALYSIS USING STIFFNESS MATRIX APPROACH

EQ. 1 $$K \cdot r = R$$

K - STIFFNESS
r - UNKNOWN DISPLACEMENTS
R - KNOWN EXTERNAL FORCES

I. DERIVE K FOR STRUCTURE
II. EXPRESS EXTERNAL LOADS IN R
III. SOLVE EQ. 1 FOR DISPLACEMENTS r
IV. FIND INTERNAL MOMENTS USING SLOPE DEFLECTION

PROCEDURE

1. DRAW STRUCTURE WITH NO EXTERNAL LOADS

2. IDENTIFY D.O.F. AND LABEL r_1, r_2 ... ETC.

3. STARTING WITH FIRST COLUMN IN K, DRAW DEFLECTED SHAPE ACCORDING TO DEFINITION OF K_{ij}

   FOR COLUMN 1, DRAW $r_1 = 1$, $r_2 = 0$, $r_3 = 0$, ...

   FOR COLUMN 2, $r_1 = 0$, $r_2 = 1$, $r_3 = 0$, ...

4. ON DEFLECTED SHAPE FIGURES, ADD A MOMENT OR FORCE AT EVERY D.O.F., ASSUMING POSITIVE DIRECTION.

5. LABEL THESE FORCES/MOMENTS ACCORDING TO DEFINITION OF K_{ij}

   FORCE D.O.F.   UNIT DEFLECTION D.O.F.

6. BREAK STRUCTURE UP INTO THE F.B.D.s NEEDED TO EXPRESS K_{ij}'s IN TERMS OF INTERNAL UNKNOWN MOMENTS M_{ij}

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PROCEDURE CONT.

7. USE SLOPE DEFLECTION EQUATION

$$M_{ij} = \frac{2EI}{L_{ij}}\left(2\theta_i + \theta_j + \frac{3D}{L_{ij}}\right)$$

AND DEFINITION OF COLUMN DISPLACEMENTS TO SOLVE FOR M_{ij} IN F.B.D.S IN STEP 6

8. SOLVE FOR K_{ij} TERMS USING EQUILIBRIUM EQS. FROM 6., & M_{ij} FROM 7.

9. MOVE TO COLUMN 2 AND REPEAT FROM STEP 3., THEN COLUMN 3, ETC. UNTIL ALL K_{ij} ARE DEFINED

MAX $i = j$ = # OF D.O.F.

* SAME PROCEDURE FOR BOTH DET. & INDET. SYSTEMS!

EXAMPLES FOLLOW

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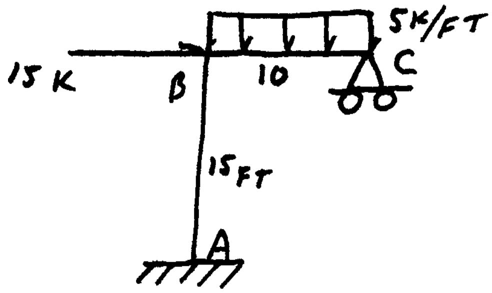
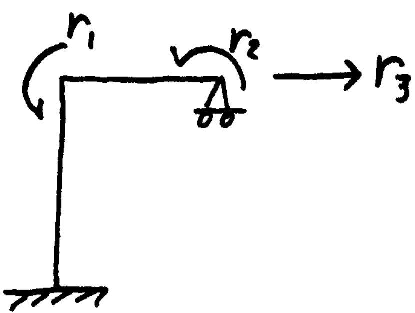
EXAMPLE 1



FIND DEFLECTIONS, ROTATIONS, REACTIONS, V/M DIAGRAMS

1. REDRAW W/OUT LOADS
2. ID + LABEL D.O.F.

$\circlearrowright +$ $\longrightarrow +$

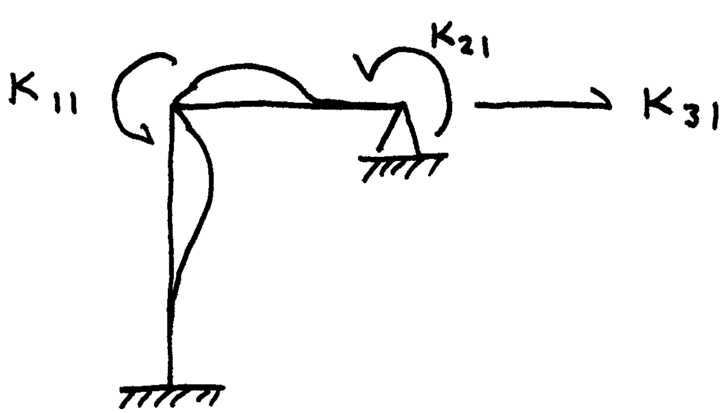


$$K = \begin{bmatrix} & ? & \end{bmatrix}_{3\times 3}$$

3. COLUMN #1 , $r_1 = 1$ $r_2 = 0$ $r_3 = 0$

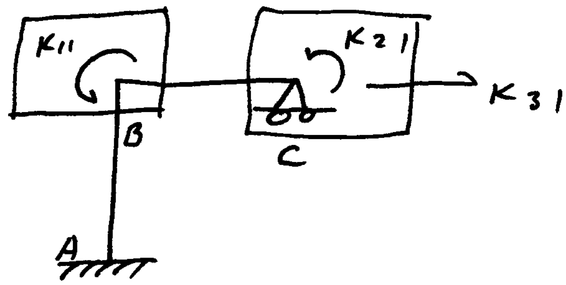
   DRAW SHAPE

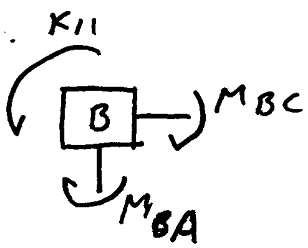
4. ADD MOMENT OR FORCE AT EACH D.O.F. IN + DIRECTION
5. LABEL FORCES ACCORDING TO K_{ij} DEFINITION

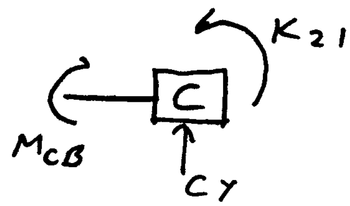


EXAMPLE 1 CONT. CES 4141

6. BREAK INTO F.B.D.s TO EXPRESS K_{11}, K_{21}, K_{31} IN TERMS OF INTERNAL MOMENTS





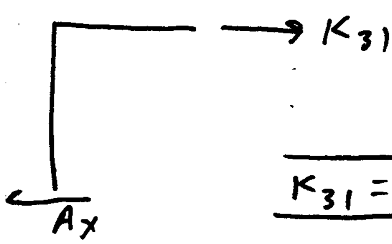


ΣM

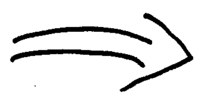
$$K_{11} = M_{BA} + M_{BC} \quad (1)$$

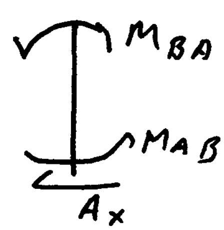
ΣM

$$K_{21} = M_{CB} \quad (2)$$



$$K_{31} = A_x$$





$$\Sigma M_B = 0 = M_{BA} + M_{AB} = A_x \cdot L_{AB}$$

$$K_{31} = \frac{M_{BA} + M_{AB}}{L_{AB}} \quad (3)$$

NOW FIND $M_{BA}, M_{BC}, M_{CB}, M_{AB}$ AND PLUG IN TO (1), (2), (3)

7. SOLVE FOR M_{ij} IN 3 EQUILIBRIUM EQS. FROM 6. USING

$$M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j + \frac{3\Delta}{L}\right)$$

&

$$r_1 = 1 \quad r_2 = 0 \quad r_3 = 0$$

FOR K_{11}

$$M_{BA} = \frac{2EI}{L_{AB}}\left(2\theta_B + \theta_A + \frac{3\Delta}{L}\right)$$

$\theta_B = r_1 = 1$

$\theta_A = 0$ NOT A D.O.F.

$\Delta = r_3 = 0$

$$M_{BA} = \frac{2EI}{L_{AB}} \cdot 2r_1 = \boxed{\frac{4EI}{L_{AB}} = M_{BA}}$$

$$M_{BC} = \frac{2EI}{L_{BC}}\left(2\theta_B + \theta_C + \frac{3\Delta}{L}\right) = \frac{2EI}{L_{BC}}\left(2\cancel{r_1}^{\,1} + \cancel{r_2}^{\,0} + 0\right)$$

$$\boxed{M_{BC} = \frac{4EI}{L_{BC}}}$$

$$\boxed{K_{11} = \frac{4EI}{L_{AB}} + \frac{4EI}{L_{BC}}}$$

FOR K_{21}

$$M_{CB} = \frac{2EI}{L_{CB}}\left(2\theta_C + \theta_B + \frac{3\Delta}{L_{BC}}\right) = \frac{2EI}{L_{CB}}\left(2\cancel{r_2}^{\,0} + \cancel{r_1}^{\,1} + 0\right)$$

$$M_{CB} = \boxed{K_{21} = \frac{2EI}{L_{CB}}}$$

FOR K_{31}

$$M_{AB} = \frac{2EI}{L_{AB}}\left(2\theta_A + \theta_B + \frac{3\Delta}{L}\right) = \frac{2EI}{L_{AB}}\left(0 + \cancel{r_1}^{\,1} + \frac{3\cancel{r_2}^{\,0}}{L_{AB}}\right)$$

$$\boxed{\begin{array}{c} M_{AB} = \dfrac{2EI}{L_{AB}} \\ \hline M_{BA} = \dfrac{4EI}{L_{AB}} \end{array}} \quad \text{FROM BEFORE}$$

$$K_{31} = \frac{M_{BA} + M_{AB}}{L_{AB}} = \boxed{\frac{6EI}{L_{AB}^2} = K_{31}}$$

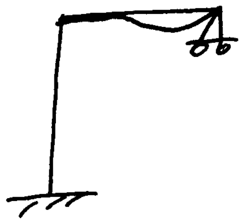
DONE WITH COLUMN 1

SO FAR WE HAVE

$$K = \begin{bmatrix} \dfrac{4EI}{L_{AB}} + \dfrac{4EI}{L_{BC}} & ?\ K_{12} & ?\ K_{13} \\ \dfrac{2EI}{L_{CB}} & ?\ K_{22} & ?\ K_{23} \\ \dfrac{6EI}{L_{AB}^2} & ?\ K_{32} & ?\ K_{33} \end{bmatrix}$$

EXAMPLE 1 CONTINUED

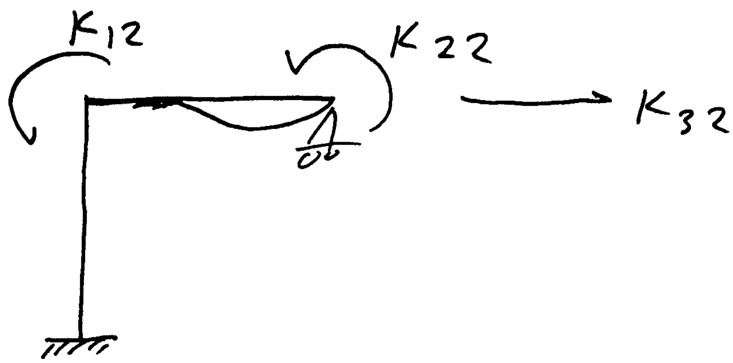
CONTINUE PROCESS FOR COLUMN 2

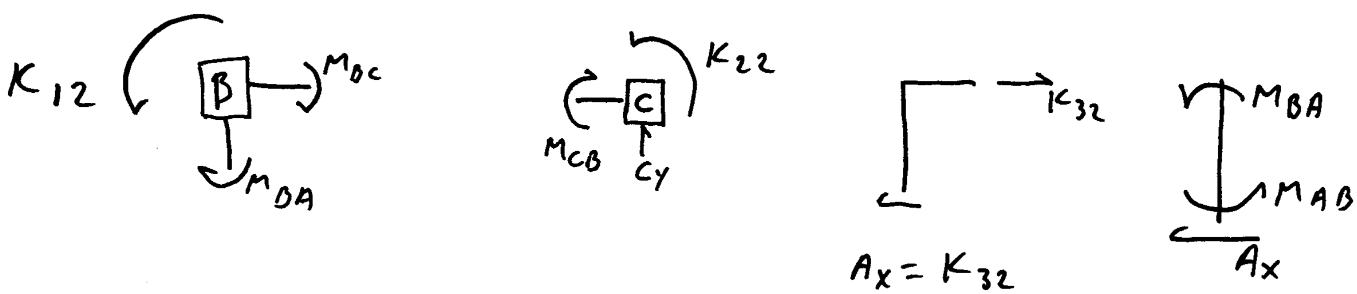


$r_1 = 0 \quad = \theta_B$

$r_2 = 1 \quad = \theta_C$

$r_3 = 0 \quad = \Delta$





$K_{12} = M_{BA} + M_{BC}$ (1)	$K_{22} = M_{CB}$ (2)	$K_{32} = \dfrac{M_{AB} + M_{BA}}{L_{AB}}$ (3)

* NOTE: FROM ONE COLUMN IN K TO NEXT, <u>SAME</u> 3 F.B.D.s ARE USED, AND <u>SAME</u> 3 EQUIL. EQS. RESULT EXCEPT K_{11} BECOMES K_{12}, K_{21} BECOMES K_{22}, $K_{31} \Rightarrow K_{32}$

ALTHOUGH F.B.D.S & EQUILIBRIUM EQS. ARE SAME AS FIRST COLUMN, WE MUST REDEFINE M_{AB}, M_{BA}, M_{BC}, M_{CB}

C1 $r_1 = 1 \quad r_2 = r_3 = 0$

C2 $r_1 = 0 \quad r_2 = 1 \quad r_3 = 0$

$$M_{AB} = \frac{2EI}{L_{AB}}\left(0 + \cancel{r_1}^{\,0} + \frac{3\cancel{r_3}^{\,0}}{L_{AB}}\right) = 0$$

$$M_{BA} = \frac{2EI}{L_{AB}}\left(2\cancel{r_1}^{\,0} + 0 + \frac{3\cancel{r_3}^{\,0}}{L_{AB}}\right) = 0$$

$$M_{BC} = \frac{2EI}{L_{BC}}\left(2\cancel{r_1}^{\,0} + \cancel{r_2}^{\,1} + 0\right) = \frac{2EI}{L_{BC}}$$

$$M_{CB} = \frac{2EI}{L_{BC}}\left(2\cancel{r_2}^{\,1} + \cancel{r_1}^{\,0} + 0\right) = \frac{4EI}{L_{BC}}$$

$$\boxed{\begin{aligned} K_{12} &= 0 + \frac{2EI}{L_{BC}} \\ K_{22} &= \frac{4EI}{L_{BC}} \\ K_{32} &= \frac{0+0}{L_{AB}} = 0 \end{aligned}}$$

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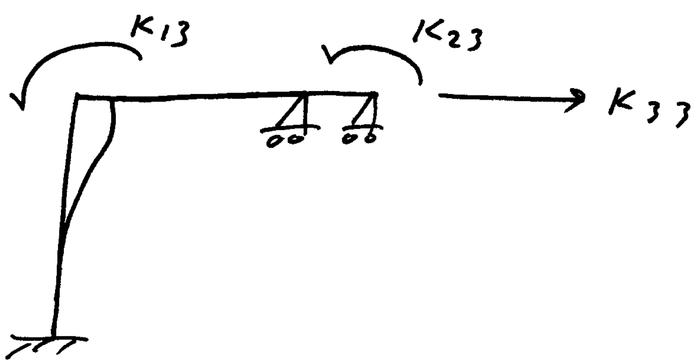
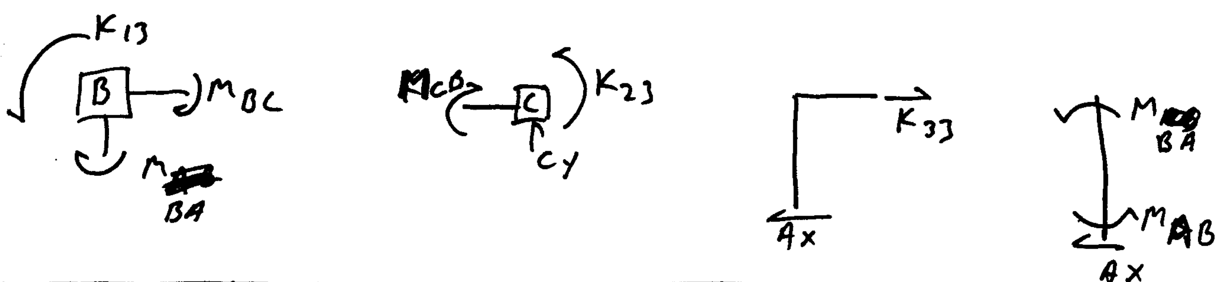
SO FAR ...
$$K = \begin{bmatrix} \frac{4EI}{L_{AB}} + \frac{4EI}{L_{BC}} & \frac{2EI}{L_{CB}} & ?\ K_{13} \\ \frac{2EI}{L_{CB}} & \frac{4EI}{L_{BC}} & ?\ K_{23} \\ \frac{6EI}{L^2_{AB}} & 0 & ?\ K_{33} \end{bmatrix}$$

FINALLY, COLUMN 3 $r_1 = 0 \quad r_2 = 0 \quad r_3 = 1$





$$\boxed{K_{13} = M_{BC} + M_{BA} \quad K_{23} = M_{CB} \quad K_{33} = \frac{M_{AB} + M_{BA}}{L_{AB}}}$$

AGAIN, SAME 3 F.B.D.S + SAME 3

EQUIL. EQS., JUST K'S CHANGE

$K_{11} = K_{12} = K_{13} = M_{BC} + M_{CB}$

$K_{21} = K_{22} = K_{23} = M_{CB}$

$K_{31} = K_{32} = K_{33} = (M_{AB} + M_{BA})/L_{AB}$

EXAMPLE 1 CONT.

REDEFINE M_{ij}'s

$$M_{AB} = \frac{2EI}{L_{AB}}\left(0 + \overset{0}{\cancel{\gamma_1}} + \frac{3\overset{1}{\cancel{\gamma_3}}}{L_{AB}}\right) = \frac{6EI}{L_{AB}^2}$$

$$M_{BA} = \frac{2EI}{L_{AB}}\left(2\overset{0}{\cancel{\gamma_1}} + 0 + \frac{3\overset{1}{\cancel{\gamma_3}}}{L_{AB}}\right) = \frac{6EI}{L_{AB}^2}$$

$$M_{BC} = \frac{2EI}{L_{BC}}\left(2\overset{0}{\cancel{\gamma_1}} + \overset{0}{\cancel{\gamma_2}} + 0\right) = 0$$

$$M_{CB} = \frac{2EI}{L_{BC}}\left(2\overset{0}{\cancel{\gamma_2}} + \overset{0}{\cancel{\gamma_1}} + 0\right) = 0$$

$$\boxed{\begin{aligned} K_{13} &= \frac{6EI}{L_{AB}^2} \\ K_{23} &= 0 \\ K_{33} &= \frac{12EI}{L_{AB}^3} \end{aligned}}$$

PUT IT ALL TOGETHER ...

$$K = \begin{bmatrix} \frac{4EI}{L_{AB}} + \frac{4EI}{L_{BC}} & \frac{2EI}{L_{CB}} & \frac{6EI}{L^2_{AB}} \\ \frac{2EI}{L_{CB}} & \frac{4EI}{L_{BC}} & 0 \\ \frac{6EI}{L^2_{AB}} & 0 & \frac{12EI}{L^3_{AB}} \end{bmatrix}$$

* A QUICK CHECK ... IS K SYMMETRIC?

$K_{ij} = K_{ji}$ FOR $i \neq j$?

$K_{12} = K_{21}$ ✓

$K_{13} = K_{31}$ ✓

$K_{32} = K_{23}$ ✓

* A QUICK CHECK ... ARE DIAGONAL TERMS + ?

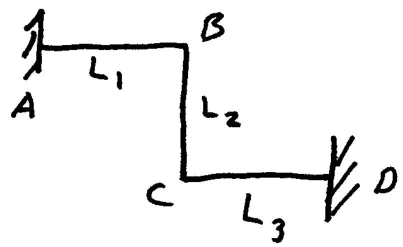
$K_{11} = +$ ✓

$K_{22} = +$ ✓

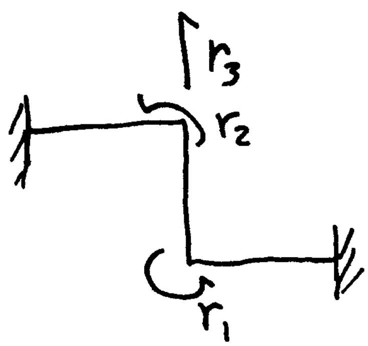
$K_{33} = +$ ✓

EXAMPLE 2

BEFORE WE GET TO LOADS AND SOLVING FOR DISPLACEMENTS, WE'LL DO ANOTHER EXAMPLE IN A MORE ORGANIZED WAY.



FIND STIFFNESS MATRIX K

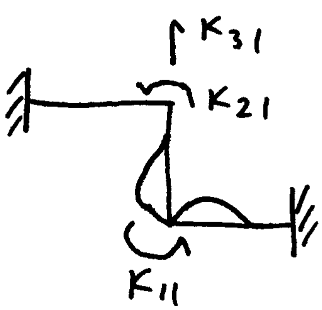


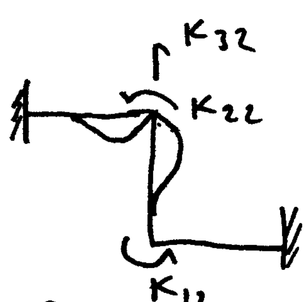
3 D.O.F.

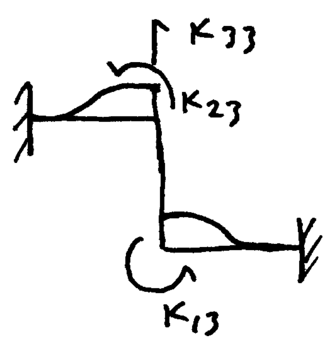
3 DEFLECTED SHAPES

C1 $r_1 = 1 \quad r_2 = r_3 = 0$

C3 $r_1 = r_2 = 0 \quad r_3 = 1$







C 2

$r_1 = r_3 = 0 \quad r_2 = 1$

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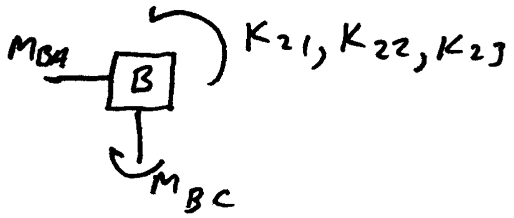
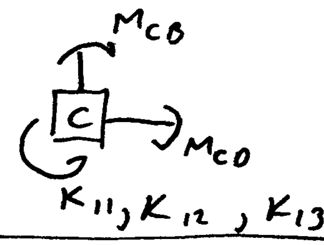
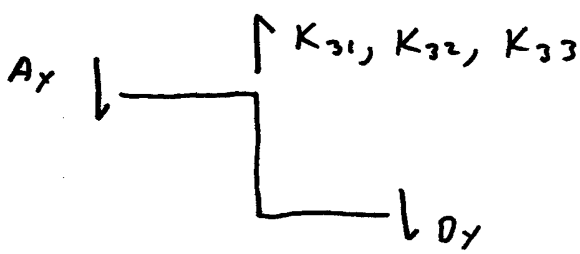
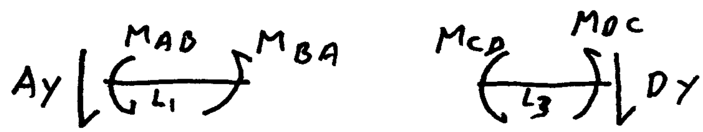
3 F.B.D.S TO GET EQUILIBRIUM EQUATIONS





$K_{21} = M_{BA} + M_{BC}$	$r_1 = 1$	$K_{11} = M_{CB} + M_{CD}$	$r_1 = 1$
$K_{22} = M_{BA} + M_{BC}$	$r_2 = 1$	$K_{12} = M_{CB} + M_{CD}$	$r_2 = 1$
$K_{23} = M_{BA} + M_{BC}$	$r_3 = 1$	$K_{13} = M_{CB} + M_{CD}$	$r_3 = 1$





$K_{31} = A_y + D_y$	$r_1 = 1$
$K_{32} = A_y + D_y$	$r_2 = 1$
$K_{33} = A_y + D_y$	$r_3 = 1$

$$A_y = \frac{-(M_{AB} + M_{BA})}{L_1} \qquad \Sigma M_B = 0$$

$$D_y = \frac{+(M_{CD} + M_{DC})}{L_3} \qquad \Sigma M_C = 0$$

$$K_{31} = \left[\frac{M_{CD} + M_{DC}}{L_3}\right] - \left[\frac{M_{AB} + M_{BA}}{L_1}\right] \qquad r_1 = 1$$

$K_{32} =$ " $\qquad r_2 = 1$

$K_{33} =$ " $\qquad r_3 = 1$

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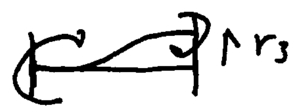
EXPAND ALL M_{ij} NEEDED

$$M_{AB} = \frac{2EI}{L_1}\left(0 + r_2 + \boxed{\frac{3(-r_3)}{L_1}}\right)$$

$\Rightarrow$ WHY $\ominus$



$$M_{BA} = \frac{2EI}{L_1}\left(2r_2 + 0 + \frac{3(-r_3)}{L_1}\right)$$

+ r_3 CAUSES $\circlearrowleft$ MOMENTS

$$M_{BC} = \frac{2EI}{L_2}\left(2r_2 + r_1 + 0\right)$$

$$M_{CB} = \frac{2EI}{L_2}\left(2r_1 + r_2 + 0\right)$$

$$M_{CD} = \frac{2EI}{L_3}\left(2r_1 + 0 + \boxed{\frac{3(+r_3)}{L_3}}\right)$$

$\Rightarrow$ WHY $\oplus$

$$M_{DC} = \frac{2EI}{L_3}\left(0 + r_1 + \frac{3(+r_3)}{L_3}\right)$$

+ r_3 CAUSES

$\circlearrowright$ MOMENTS

NOW PLUG M_{ij} INTO EQUILIBRIUM EQS.

$r_1 = 1 \quad r_2 = 0 \quad r_3 = 0$

$$K_{11} = M_{CB} + M_{CD} = \frac{4EI}{L_2} + \frac{4EI}{L_3} = K_{11}$$

$$K_{21} = M_{BA} + M_{BC} = \frac{2EI}{L_2} = K_{21}$$

$$K_{31} = \frac{M_{CD} + M_{DC}}{L_3} - \frac{M_{AB} + M_{BA}}{L_1} = \frac{\frac{4EI}{L_3} + \frac{2EI}{L_3}}{L_3} = K_{31}$$

$r_1 = 0 \quad r_2 = 1 \quad r_3 = 0$

$$K_{12} = M_{CB} + M_{CD} = \frac{2EI}{L_2} = K_{12}$$

$$K_{22} = M_{BA} + M_{BC} = \frac{4EI}{L_1} + \frac{4EI}{L_2} = K_{22}$$

$$K_{32} = \frac{M_{CD} + M_{DC}}{L_3} - \frac{M_{AB} + M_{BA}}{L_1} = 0 - \left(\frac{\frac{2EI}{L_1} + \frac{4EI}{L_1}}{L_1} \right) = K_{32}$$

$r_1 = r_2 = 0 \quad r_3 = 1$

$$K_{13} = M_{CB} + M_{CD} = \frac{6EI}{L_3^2} = K_{13}$$

$$K_{23} = M_{BA} + M_{BC} = -\frac{6EI}{L_1^2} = K_{23}$$

$$K_{33} = \frac{M_{CD} + M_{DC}}{L_3} - \frac{M_{AB} + M_{BA}}{L_1} = \underline{\frac{12EI}{L_3^2} \Big/ L_3 + \frac{12EI}{L_1^2} \Big/ L_1}$$

PUT RESULTS IN MATRIX

$$K = \begin{bmatrix} \frac{4EI}{L_2} + \frac{4EI}{L_3} & \frac{2EI}{L_2} & \frac{6EI}{L_3^2} \\ \frac{2EI}{L_2} & \frac{4EI}{L_1} + \frac{4EI}{L_2} & -\frac{6EI}{L_1^2} \\ \frac{6EI}{L_3^2} & -\frac{6EI}{L_1^2} & \frac{12EI}{L_1^3} + \frac{12EI}{L_3^3} \end{bmatrix}$$

CHECK FOR SYMMETRY

$K_{12} = K_{21}$ ✓

$K_{23} = K_{32}$ ✓

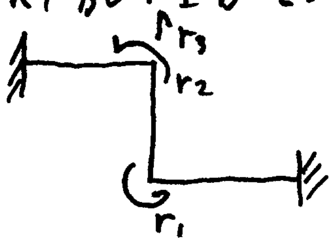
$K_{13} = K_{13}$ ✓

CHECK DIAGONALS POSITIVE

K_{11}, K_{22}, K_{33} ALL $+$ ✓

LOADS

- POINT LOADS AND POINT MOMENTS
- DISTRIBUTED LOADS



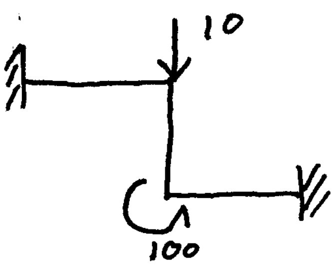
$$\underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}_{K \quad 3\times 3} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

r = DISPLACEMENTS AT D.O.F.

R = LOADS AT D.O.F.

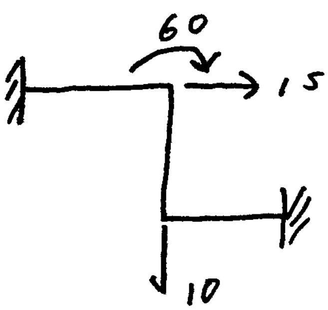
SOLVING $K \cdot r = R$

LOADS ONLY SHOW UP IN EQUATION AT D.O.F.



$$R = \begin{bmatrix} 100 \\ 0 \\ -10 \end{bmatrix}$$

↓10 OPPOSITE DIRECTION FROM ↑r_3

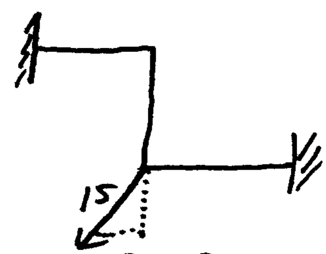


$$R = \begin{bmatrix} 0 \\ -60 \\ -10 \end{bmatrix}$$

→15 NOT AT A D.O.F. (IGNORING AXIAL)

↑r_3 ↻r_2

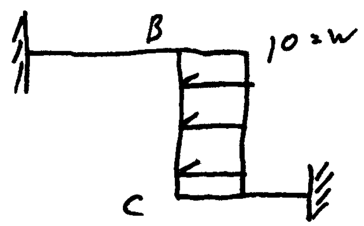
↓10 AT B OR C IS SAME



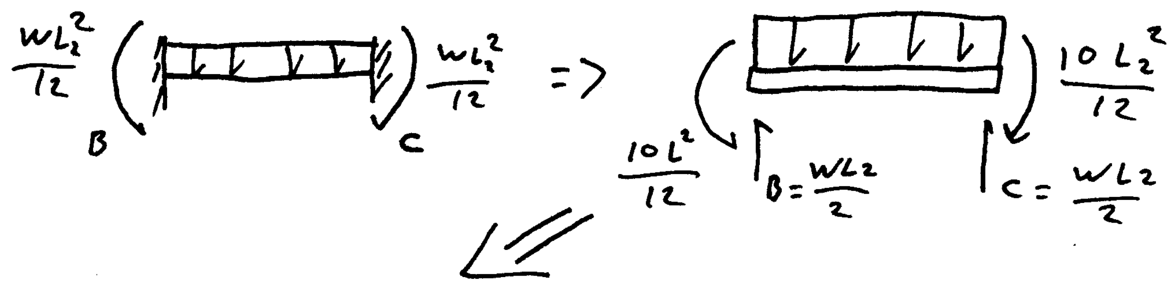
3·4·5 TRIANGLE

$$R = \begin{bmatrix} 0 \\ 0 \\ -15\left(\frac{4}{5}\right) \end{bmatrix}$$

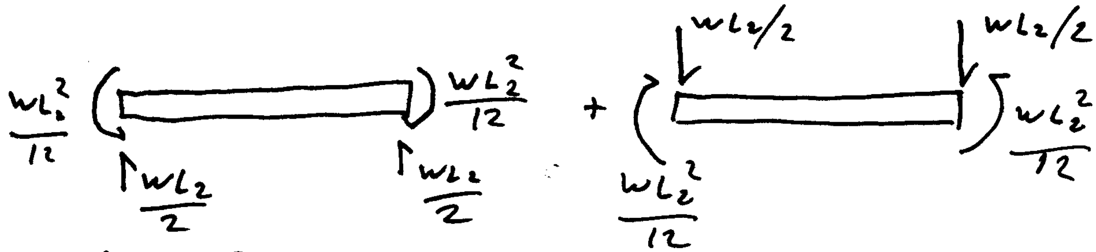
LOADS BETWEEN ENDS OF MEMBERS
MUST BE RESOLVED TO
<u>EQUIVALENT NODAL LOADS</u>
USING F. E. M.



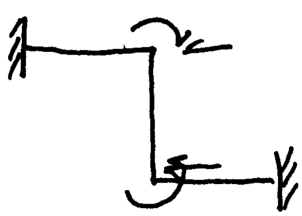
SOLVE FOR SHEAR REACTION



GET RID OF DIST. LOAD
REPLACE WITH NODAL
LOADS THAT REQUIRE
SAME REACTIONS



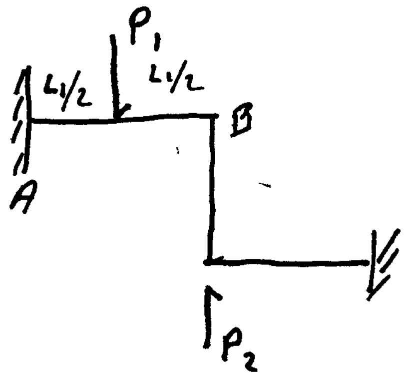
<u>NOW</u>

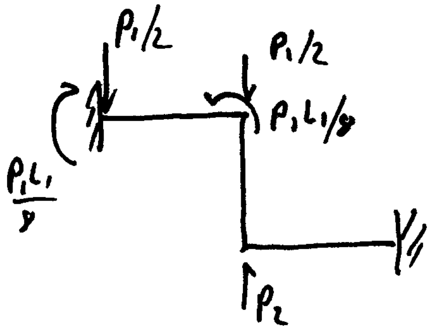


$$\Rightarrow \quad R = \begin{bmatrix} wL_2^2/12 \\ -wL_2^2/12 \\ 0 \end{bmatrix}$$

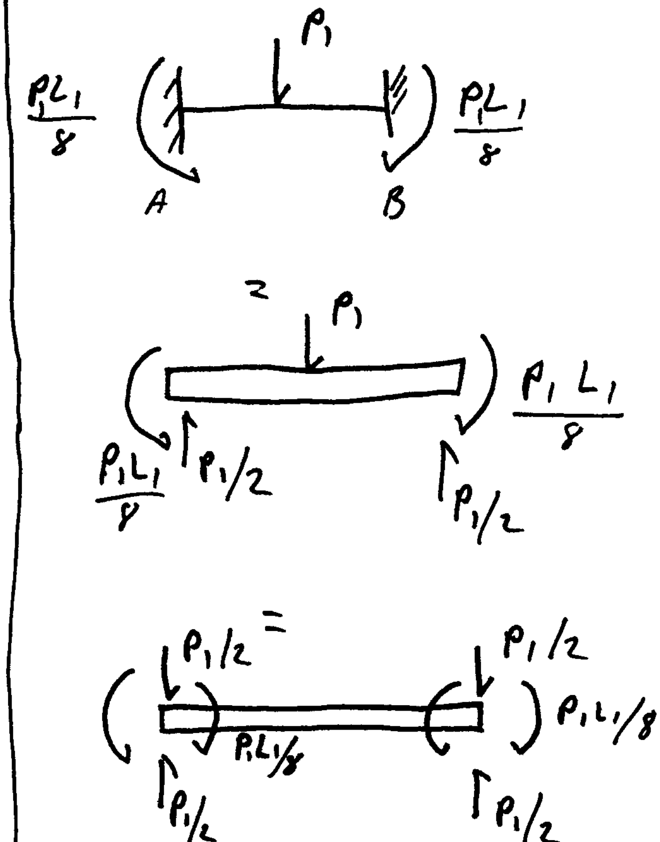
EQUIVALENT NODAL LOADS

LOADS CONT. CES 4141 20

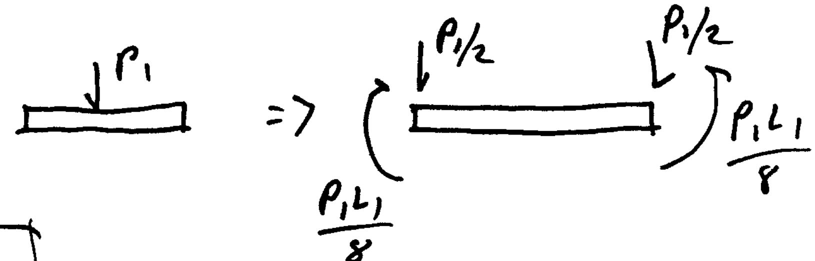




EQUIV. NODAL LOADS



∴ EQUIVALENT NODAL LOADS



$$\boxed{R = \begin{bmatrix} 0 \\ P_1L_1/8 \\ -P_1/2 + P_2 \end{bmatrix}}$$

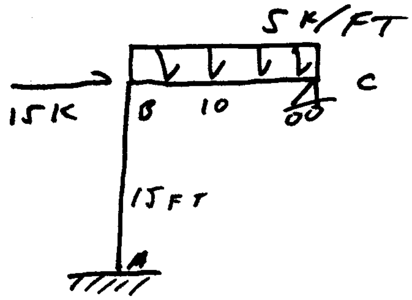
SUPERIMPOSE MULTIPLE LOADS AT SAME D.O.F.

$$K \cdot r = R$$

EXAMPLE #1 w/LOADS

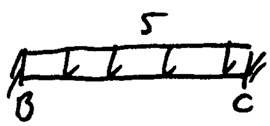
BACK TO EXAMPLE 1

ADD LOADS

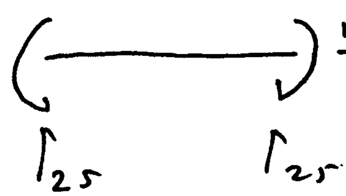


$$\underbrace{\begin{bmatrix} \frac{4EI}{L_{AB}} + \frac{4EI}{L_{BC}} & \frac{2EI}{L_{CB}} & \frac{6EI}{L_{AB}^2} \\ \frac{2EI}{L_{CB}} & \frac{4EI}{L_{BC}} & 0 \\ \frac{6EI}{L_{AB}^2} & 0 & \frac{12EI}{L_{AB}^3} \end{bmatrix}}_{K}$$

GET R

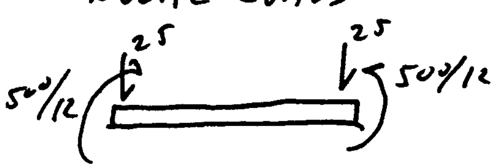


REACTIONS



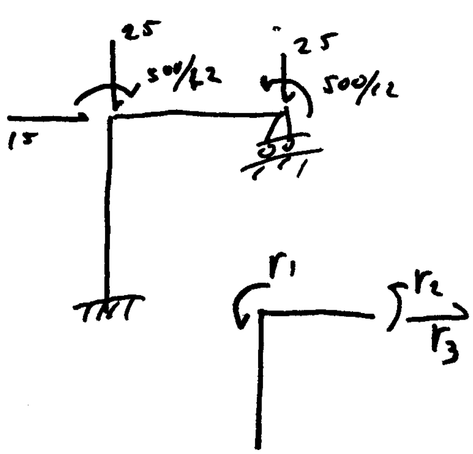
$\frac{wL^2}{12} = \frac{500}{12}$

EQUIVALENT NODAL LOADS



APPLY EQUAL & OPPOSITE OF REACTIONAL LOADS

NEW PICTURE



$$R = \begin{bmatrix} -500/12 \\ 500/12 \\ 15 \end{bmatrix}$$

$\downarrow 25$ LOADS NOT AT A D.O.F.

NOW LET'S ADD #'S IN K & SOLVE

$E = 29000$ KSI
$I = 400$ IN^4
$L_{AB} = 15$ FT
$L_{BC} = 10$ FT

ALL CONVERTED TO IN, K

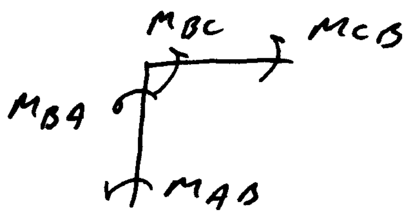
$$\begin{bmatrix} 6.44 & 1.93 & 0.0215 \\ 1.93 & 3.866 & 0 \\ 0.0215 & 0 & .0002 \end{bmatrix} \times 10^5 \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} -500 \\ 500 \\ 15 \end{bmatrix}$$

USE H.P. OR GAUSS ELIMINATION

$$\boxed{\begin{aligned} r_1 &= -.0059 \text{ RAD} \\ r_2 &= .0043 \text{ RAD} \\ r_3 &= 1.1617 \text{ IN} \end{aligned}}$$

NOW WE HAVE r ALL DISPLACEMENTS

WE CAN GO GET INTERNAL MOMENTS
& REACTIONS



$$M_{AB} = \frac{2EI}{L_{AB}}\left(2\cancelto{0}{\theta_A} + \cancelto{-.0059}{\theta_B} + \frac{3\cancelto{1.1617}{\Delta}}{L}\right) + 0$$

$$M_{BA} = \frac{2EI}{L_{AB}}\left(2(-.0059) + 0 + \frac{3(1.1617)}{L_{AB}}\right) + 0$$

$$M_{BC} = \frac{2EI}{L_{BC}}\left(2(-.0059) + .0043\right) + 500 \text{ K.IN}$$

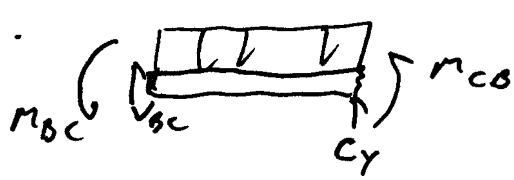
$$M_{CB} = \frac{2EI}{L_{BC}}\left(2(.0043) - .0059\right) - 500 \text{ K.IN}$$

GET #'S,

$M_{AB} = M_A$ REACTION

FIND INTERNAL SHEARS

E.G.



ETC....